Statistical Standard, Methodology and Application in Data Management and Usage

Optimal Filter to Analyze the Topological Network of Stocks Market

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Message to the audiences

This talk is to introduce a new theory to analyze the behavior of stocks market. It is an improvement of our previous theory published in **Djauhari and Gan (2015)** and appeared in the journal of Econophysics "*Physica A: Statistical Mechanics and its applications*" **(Q2, IF: 2.243, H Index: 126, More than 200 reads in RG)**

Two examples (NYSE and KLSE) will be presented to illustrate its advantages.

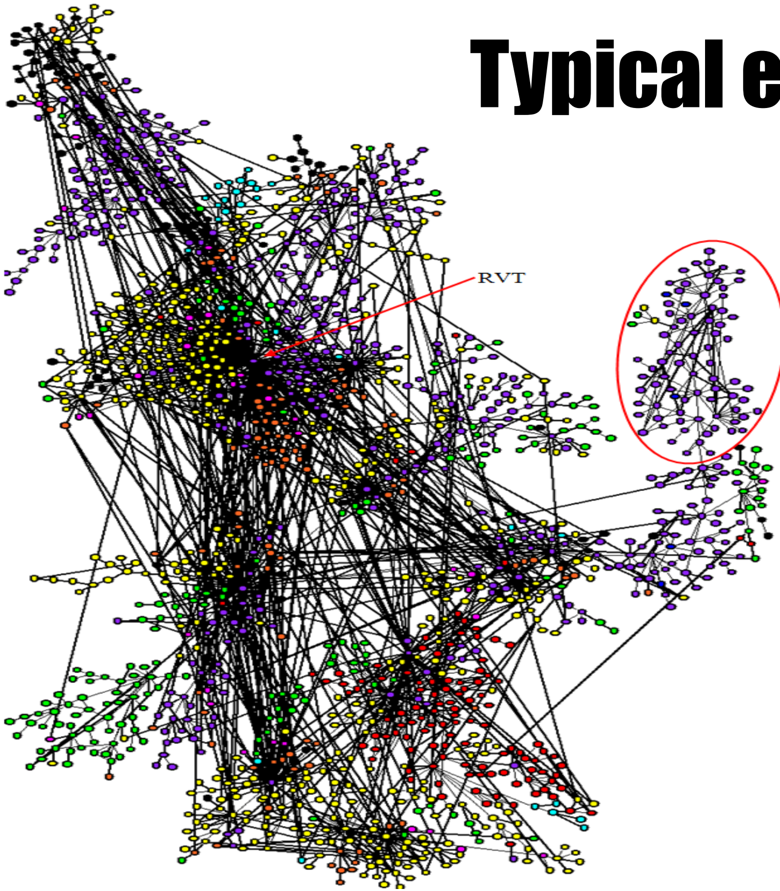
Highlights

- Stocks market is a complex system (consisting of the stocks and their relationships/interactions). By using GTA, stocks market behavior is represented in the form of a complex network.
- The larger the number of stocks, the higher the complexity level of market behavior. To reduce the complexity level, the network is filtered.
- The most adopted filter is not optimal to quantify and visualize the most important transmission of stock's behavior to the others'.
- A theory of optimal filter is then introduced and a method to find it is developed.
- Two examples will be presented; one comes from NYSE and the other one from KLSE.

Problem & Idea

- We show that the most adopted filter, MST, is not optimal. Therefore, MST-based network topology provides a misleading economic interpretation.
- In this talk, we introduce a theory of optimal MST and then develop a method to find it.
- To implement this theory, we use the notion of the forest of all MSTs (Forest, in short).
- Once an optimal MST has been found, the economic interpretation of the corresponding network topology is derived by means of usual tools such as network centralities and the power-law of degree distribution.

Typical example



Complex network of 1515 stocks traded at NYSE from 2005 – 2014. The links represent their inter-correlations (data were collected from www.nasdaq.com).

How to find an optimal network topology?

Main advantages of Forest

Why we use Forest? As we will show,

- It simplifies the process to find an optimal MST and lead us to a uniqueness theorem of MST and to a robust filter.
- It leads to an efficient algorithm to find MST; more efficient than Kruskal's algorithm and Prim's that usually used in stocks network analysis.
- Forest provides a direct computation of SDU.

NOTE: If MST is to visualize the most important transmission of a stock's behavior to the others', SDU is to represent the hierarchical taxonomy of the evolution of the most interacted stocks.

Unpleasant property of MST-based filter

Almost certain that, in practice, a degeneracy of correlation coefficients is present. This makes,

- MST not unique. It is then hard to believe that the MST issued from the most widely used algorithm is optimal in representing the market behavior.
- It depends on nodes ordering.

To handle this problem, in the next slides we introduce a theory of optimal MST.

A theory of optimal MST

I. Criteria

Let F denote the Forest in our study. We say that M^*, an MST in F, is optimal if it satisfies these three conditions,

1. $D(M^*) = \min_{M \text{ in } F} \{D(M)\}$ where $D(M)$ is the diameter of M.
2. $SPL_G(M^*) = \min_{M \text{ in } F} \{SPL_G(M)\}$ where

   $$SPL_G(M) = \sum_{i \text{ in } M} \sum_{j \text{ in } M} PL_{ij} ,$$

   and PL_{ij} is the length of the path between stocks i and j in M. In other words, $SPL_G(M)$ is the total path lengths in M.
3. $SPL_S(M^*) = \min_{M \text{ in } F} \{SPL_S(M)\}$ where

   $$SPL_S(M) = \sum_{k=1,\ldots,K} \sum_{i \text{ in } B(k)} \sum_{j \text{ in } B(k)} PL_{ij}$$

   and K is the number of business sectors, and $B(k)$ is the smallest sub-tree of M consisting of all stocks in the k-th business sector.

NOTE:

These criteria are simpler than those proposed in **Djauhari and Gan (2015)** because we omit the condition on the number of leaves.

We can show that the number of leaves in *F* and that in any MST are the same.

NOTE:

For computation purpose, we can show that

$$SPL_G(M) = (n-1)\sum_{i \text{ in } M} 1/cc(i)$$

where $cc(i)$ is the closeness centrality score of the stock i (Borgatti (2005)).

As the transmission of stock's behavior to the others is fast in an optimal MST, the value of $SPL_G(M^*)$ is then minimum.

This criterion reflects the connectedness of stocks in the same business sector; they are strongly connected. From economic point of view, they are homogeneous and tend to be clustered. Therefore, in an optimal MST, its value is minimal.

II. Selection Procedure

- Construction of the Forest
- Construction of Optimal MST

Construction of the Forest

Let D be the distance matrix related to the correlation matrix as defined in Mantegna and Stanley (2000). We construct the sequence $D, D^2, D^4, D^8, \ldots$ where matrix multiplication is defined in the usual sense but multiplication and summation of two real numbers a and b are defined as $\max\{a,b\}$ and $\min\{a,b\}$.

This sequence has a limit that can be achieved in less than $1.4427\ln(n)$ iterations (Djauhari (2012)).

Let $d^+(i,j)$ be the (i,j)-th element of this limit. If Δ is the adjacency matrix of the Forest, then its (i,j)-th element is,

$$\delta(i,j) = \begin{cases} 1; d(i,j) - d^+(i,j) = 0 \text{ and } i \neq j \\ 0; d(i,j) - d^+(i,j) \neq 0 \text{ or } i = j \end{cases}$$

where $d(i,j)$ is the (i,j)-th element of D.

NOTE (Uniqueness theorem of MST):

As an immediate result of this construction, we have a uniqueness theorem of MST: "*If N is the sum of all elements of* Δ, *then MST is unique if and only if* $N = 2(n-1)$." This theorem is important to test whether MST in the network is unique or not.

Construction of Optimal MST

- An important property of *F* is that an MST is optimal in the market if and only if it is optimal in *F*.
- Let us remove all leaves from *F* and denote *H* the remaining sub-graph. Evidently, *H* could contain no leaf or at least one. In the latter case, we repeat the process of leaves removal until we get the sub-graph *H** consisting no leaf.
- If we need *m* repetitions and *M* is an MST in *H**, then the union of *M* and all removed leaves in the *m*-th repetition and all removed leaves in the -th repetition and … all removed leaves in the 1st removal, is an MST of the stocks network.

- As a result, an optimal MST can simply be selected heuristically from *H**.

Case of NYSE

- The 100 most traded stocks at NYSE during the whole year 2012 are analyzed based on Closing Price. Only n = 98 stocks are further analyzed due to the completeness of data.
- The daily data were downloaded from http://finance.yahoo.com. and www.nyse.com/about/listed/nyid_components.shtml.
- The correlation matrix is defined in such a way that the *i*-th row (and column) corresponds to the *i*-th stock listed in these links (accessed on July 25, 2013).
- MST issued from Kruskal's algorithm (Kruskal network, in brief) and the Forest are presented in Figure 1 and 2.

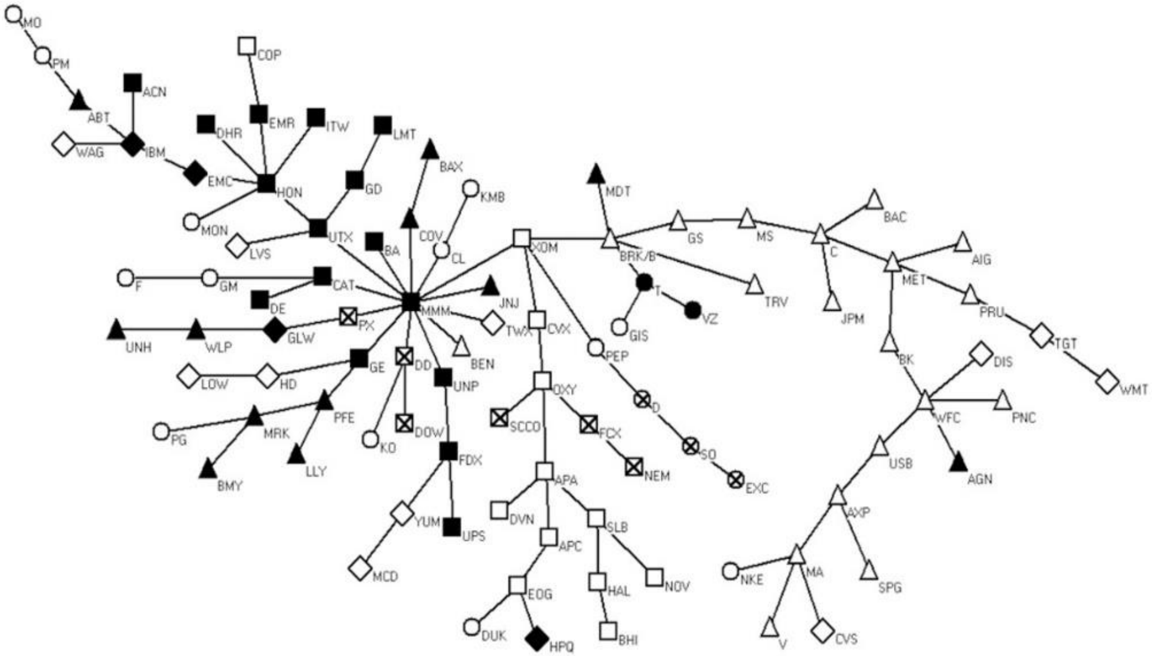


Figure 1: Kruskal's network topology of 98 most traded stocks at NYSE

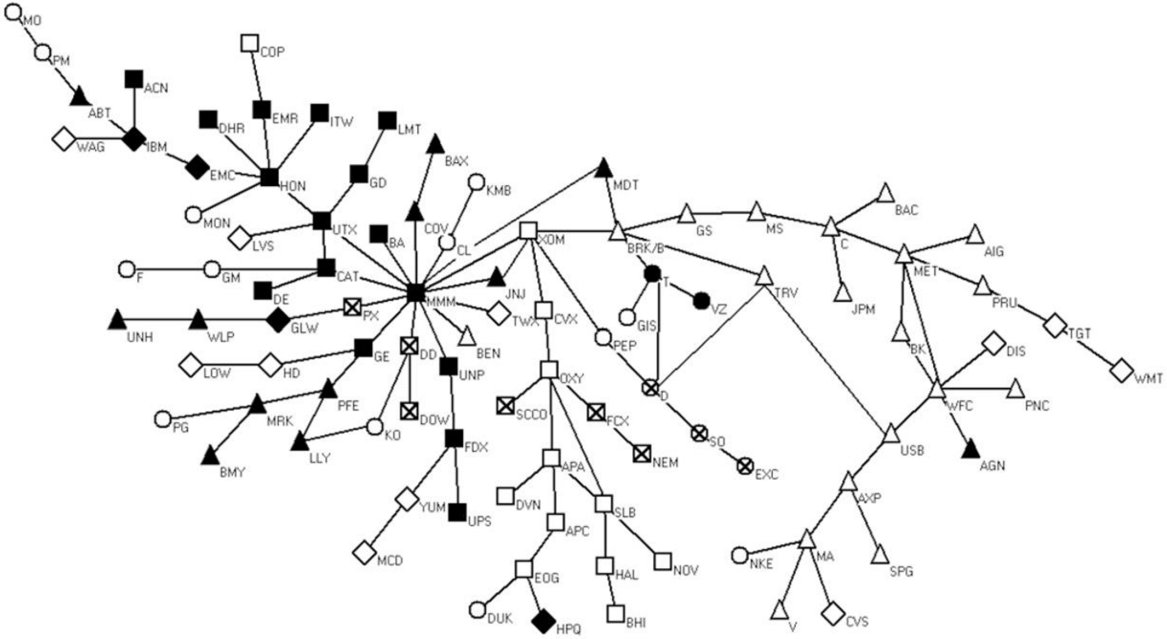


Figure 2: Forest of 98 most traded stocks at NYSE

NOTE:

Figure 2 indicates that it is hard to believe that Kruskal's network (MST-based filter) is optimal. In fact, as we will show in Figure 3, the sub-graph H^* of Forest contains many circuits. Therefore, there are so many MSTs therein.

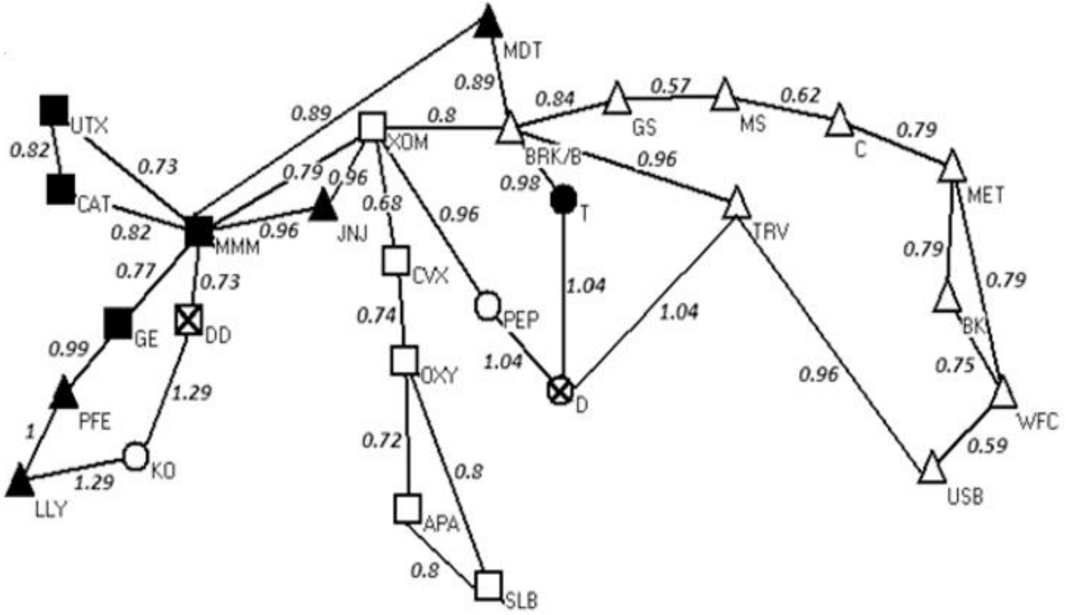


Figure 3: H* - The largest sub-graph of Forest without leaf

NOTE:

After a deep study on H^*, we find that the Kruskal's MST in H^* which leads to an optimal MST in the network is as showed in Figure 4.

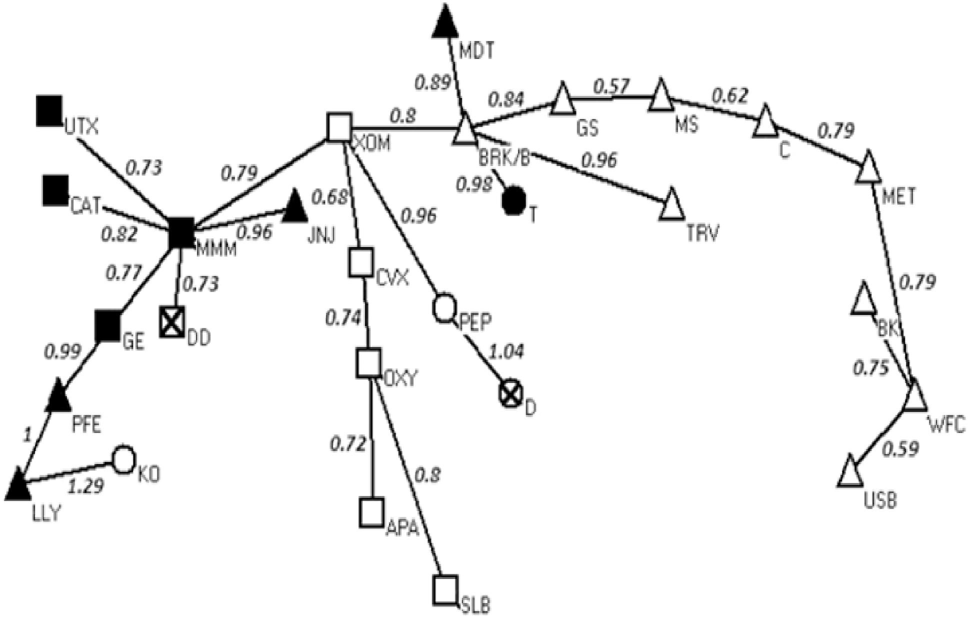


Figure 4: Kruskal's MST of H* leading to optimal network topology

Optimal network topology of NYSE

- The union of MST of H^* in Figure 4 and all removed leaves in the last repetition and all removed leaves in the second last repetition and … all removed leaves in the 1^{st} removal, is an optimal MST that we are looking for.

- The result is in Figure 5.

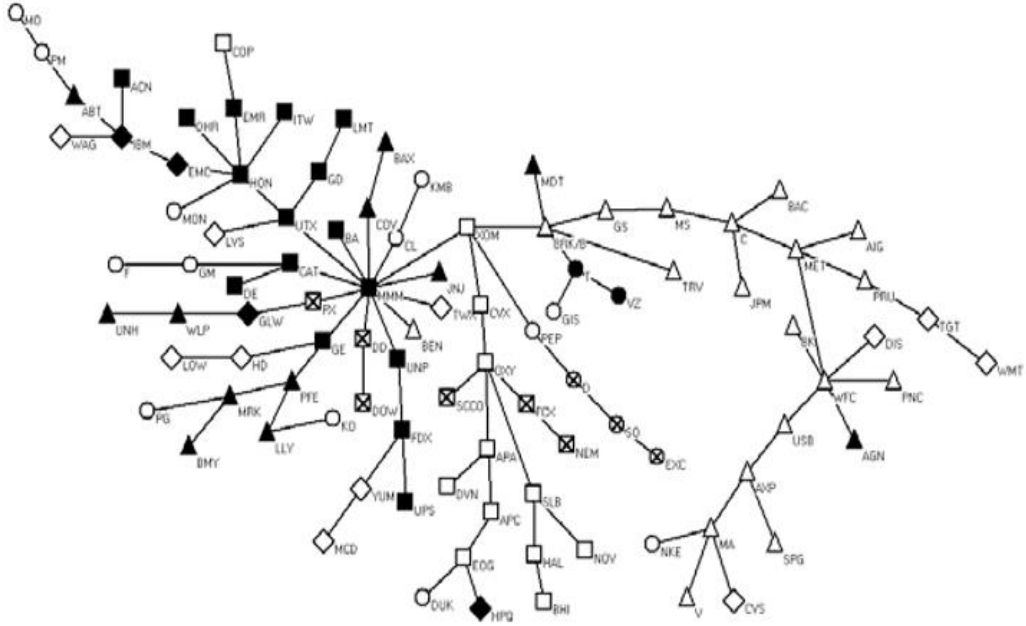


Figure 5: Optimal network topology

NOTE:

For comparison study, we also present the **worst network** in Figure 6.

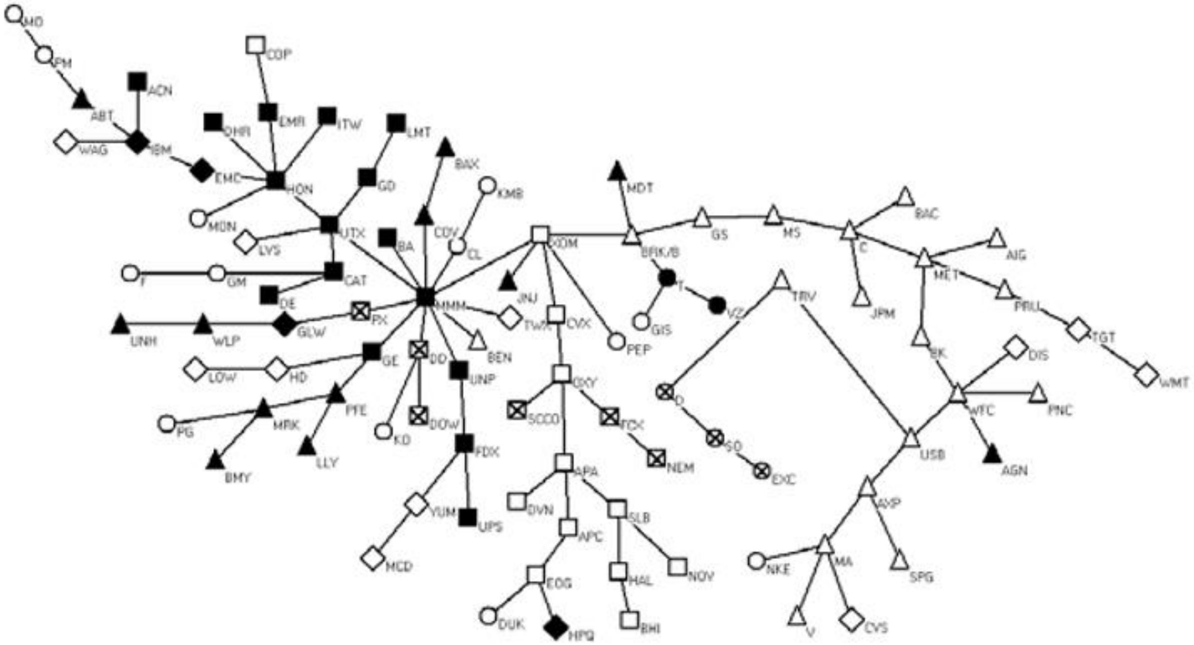


Figure 6: Worst network topology

Network topology property

Table 1: The characteristics of network topology

Characteristic	Network topology		
	Optimal	Kruskal	Worst
Diameter	18	19	20
SPL_G	67058	69094	73314
SPL_S	5780	5956	5974

Table 2: The characteristics of power-law

Characteristic	Network topology		
	Optimal	Kruskal	Worst
Constant c	0.5852	0.6035	0.6715
Exponent γ	1.7102	1.7822	1.8619

NOTE:

These two tables show that the theory of optimal MST introduced here finds its justification. All characteristics (diameter, SPL_G, SPL_S, c and γ) of optimal network have smallest value.

Case of KLSE

- We analyze 100 most traded stocks at KLSE from January 2007 to January 2009. Here also, the data are about Closing Price.
- Only 90 stocks are further analyzed. The data for the other 10 stocks are not complete.
- MST issued from Kruskal's algorithm is in Figure 7. The nodes are colored according to their business sector; blue (*trading and services*), black (*technology*), brown (*properties*), cyan (*industrial products*), olive green (*finance*), pink (*plantation*), purple (*IPC*), red (*construction*), and yellow (*consumer products*).

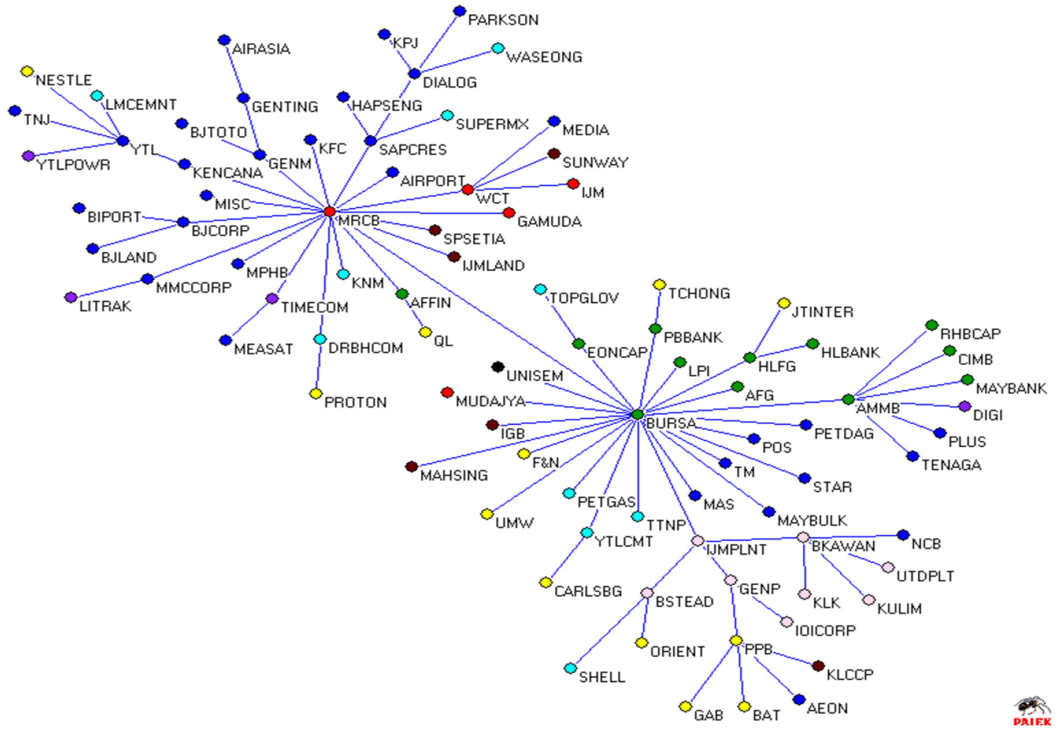


Figure 7: Kruskal's network topology of 90 most traded stocks at KLSE

NOTE:

- At a glance, in that figure, two big clusters reveal. The upper cluster is led by MRCB (Malaysian Resources Corporation Berhad) in the *Construction* sector.
- Meanwhile, the lower one is led by BURSA (Bursa Malaysia Berhad) in the *Finance* sector.
- Unfortunately, it can be shown (Show!) a degeneracy of the correlation coefficients is present which means that MST is not unique. Thus, it is questionable that the network in this figure is optimal.
- By using the same analysis as in NYSE case, the readers are invited to find an optimal network for KLSE.

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THANK YOU FOR YOUR INTEREST

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